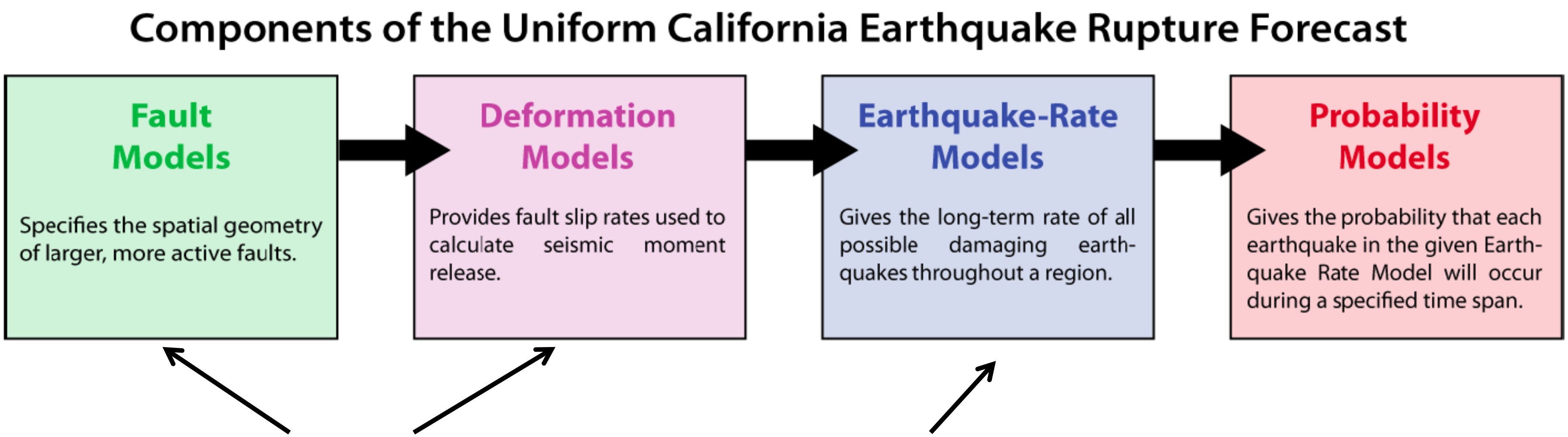
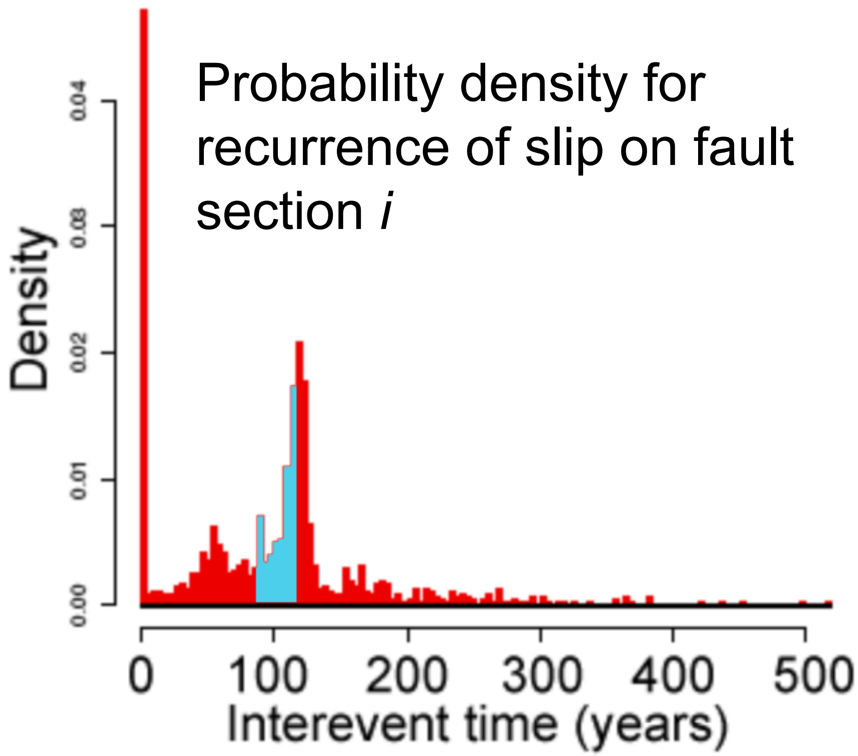
Use of simulations for long-term assessment of earthquake probabilities



Inputs to simulations

Use tuned earthquake simulations to generate earthquake rate models

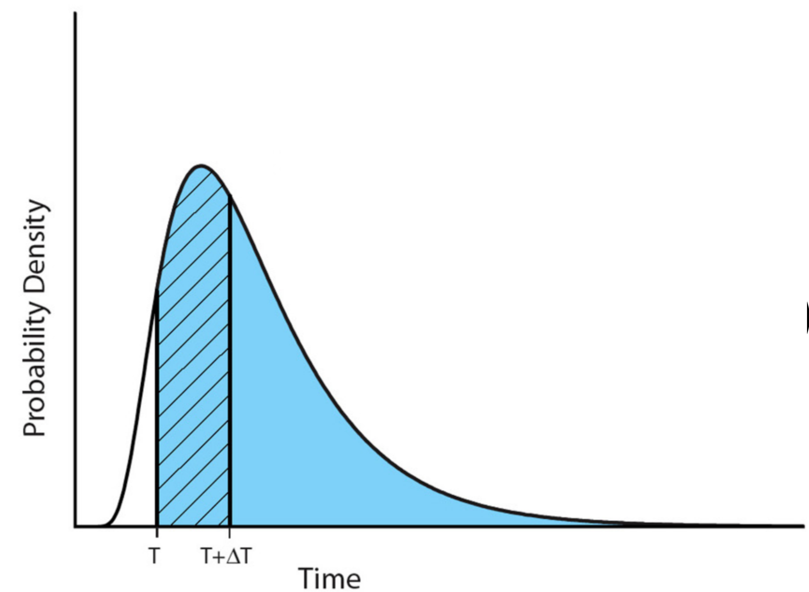
Conditional probabilities at fault sub-section *i*



CP_i = conditional probability of event $M \geq M_{min}$ in interval ΔT on section *i*, given elapsed time T_i

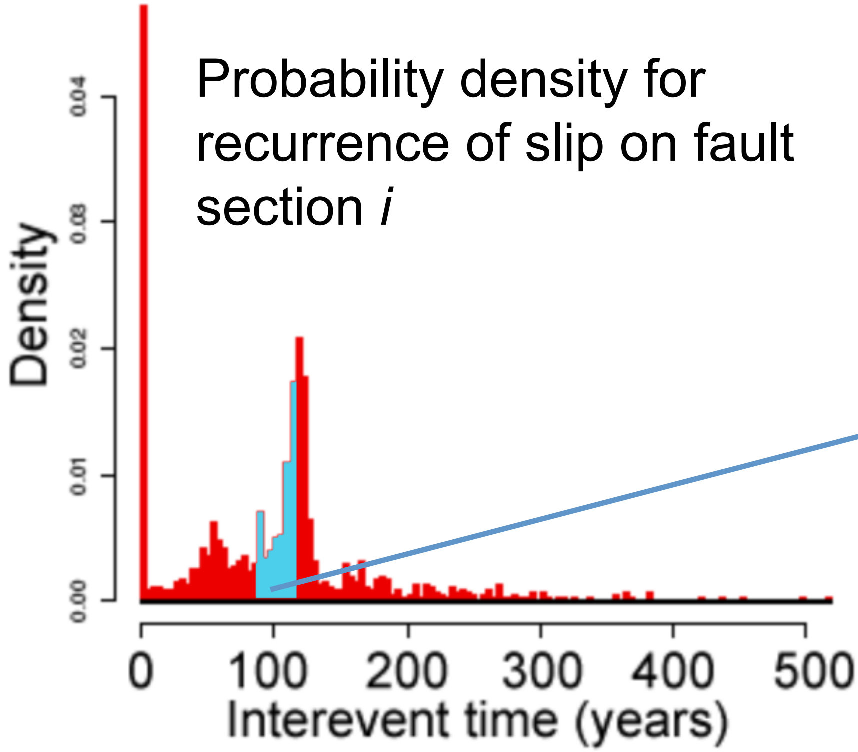
$$F(T) = \int_T^{\infty} f(t)dt$$

$$P(T \leq t \leq T + \Delta T \mid t > T) = \frac{F(T) - F(T + \Delta T)}{F(T)}$$

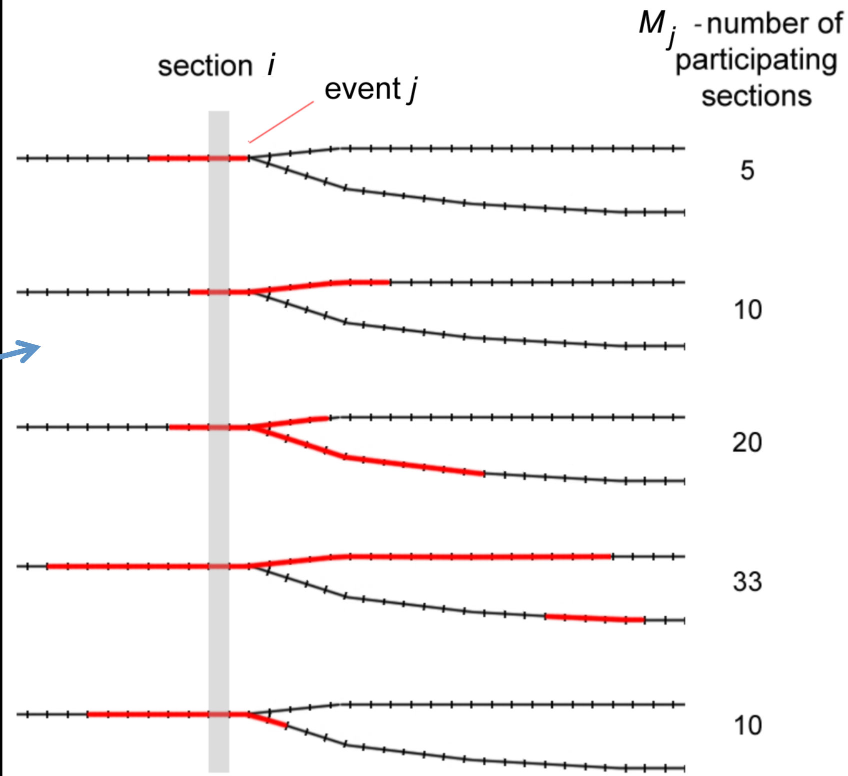


The events in the interval ΔT of the pdf provide a sample of possible events that may occur in the future.

Probability density for recurrence of slip on fault section *i*



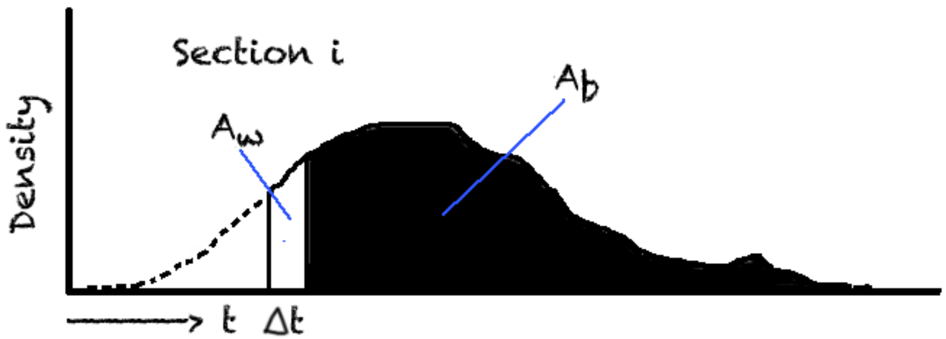
CP_i = conditional probability of event $M \geq M_{min}$ in interval ΔT on section *i*, given elapsed time T_i



Sub-catalog of *n* events for a section that occur in the interval ΔT used to determine CP_i

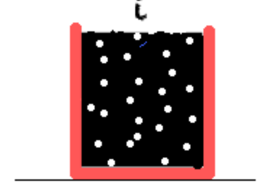
Conditional probability of an event in the interval Δt, given elapsed time t with no event

$$CP_i = A_w/(A_w + A_b)$$



This can be represented by drawing from a jar of white and black marbles, where events in the Δt sub-catalog are represented by white marbles and the events in the >(t+Δt) catalog are represented by black marbles

$$CP_i = \frac{\sum(\text{white marbles})}{n_t}$$



where nt is the sum of white and black marbles ($n_t = n_w + n_b$) and $n_w/n_b = A_w/A_b$. Similarly, the conditional probability can be representedby as the sum of the event probabilities

$$CP_i = \sum_j \delta_{ij}\, prob_j$$

$\delta_{ij} = 0$ event j not in the section i Δt catalog

$\delta_{ij} = 1$ event j in the section i Δt catalog

Simulated annealing

$$CP_i = \sum_j \delta_{ij}\, prob_j$$

Because the number of event probabilites, probj, greatly exceeds the number of sections, this set of equations for probj is under-determined. We use simulated annealing to find optimal solutions for probj.

Temperature is represented as Gaussian noise on probj and follows a cooling schedule where time advances in discrete steps

$$T = A\exp(-Bt) \longrightarrow std(\Delta p) = A\exp(-BN_{step})$$

Energy state is represented as the RMS error on the trial solutions

$$Error = \sqrt{\sum_i \{(CP_i - \sum_j \delta_{ij}\, prob_j)^2\}}$$

The set of trial solutions for probj is accepted and Nstep advances if the trial solution Error is less than current Error

For the trial simulations

N_{Limit}=250,000 A=0.03 B=10/N_{Limit}

Simulated annealing

$$CP_i = \sum_j \delta_{ij} \, prob_j$$

To start the simulated annealing process a good first estimate of the event probabilities is found assuming the probabilities of events in the Δt catalog of some section i contribute equally to the section conditional probability. For event j

$$prob_j = CP_i / n, \quad j = 1 \ldots n, \; n = \text{total number of events in the } \Delta t \text{ sub-catalog}$$

However, if event j appears in other section Δt catalogs, it will generally have different values.

The average of the section probabilities is used to start the simulation.

Simulated annealing to invert for event probabilities $prob_j$

Section probabilities

Event probabilities

$$CP_i = \sum_{j=1}^{J} \delta_{ij} w_{ij} prob_j$$

i – index on fault sections

j – index on earthquake events that appear in the Δt sub-catalogs

δ_{ij} – participation matrix. Takes value of 1 for section numbers *i* that participate in earthquake *j.* Otherwise value = 0.

w_{ij} – linear weighting function inversely proportional to the normalized number of sections that have event *i* in the Δt sub-catalogs

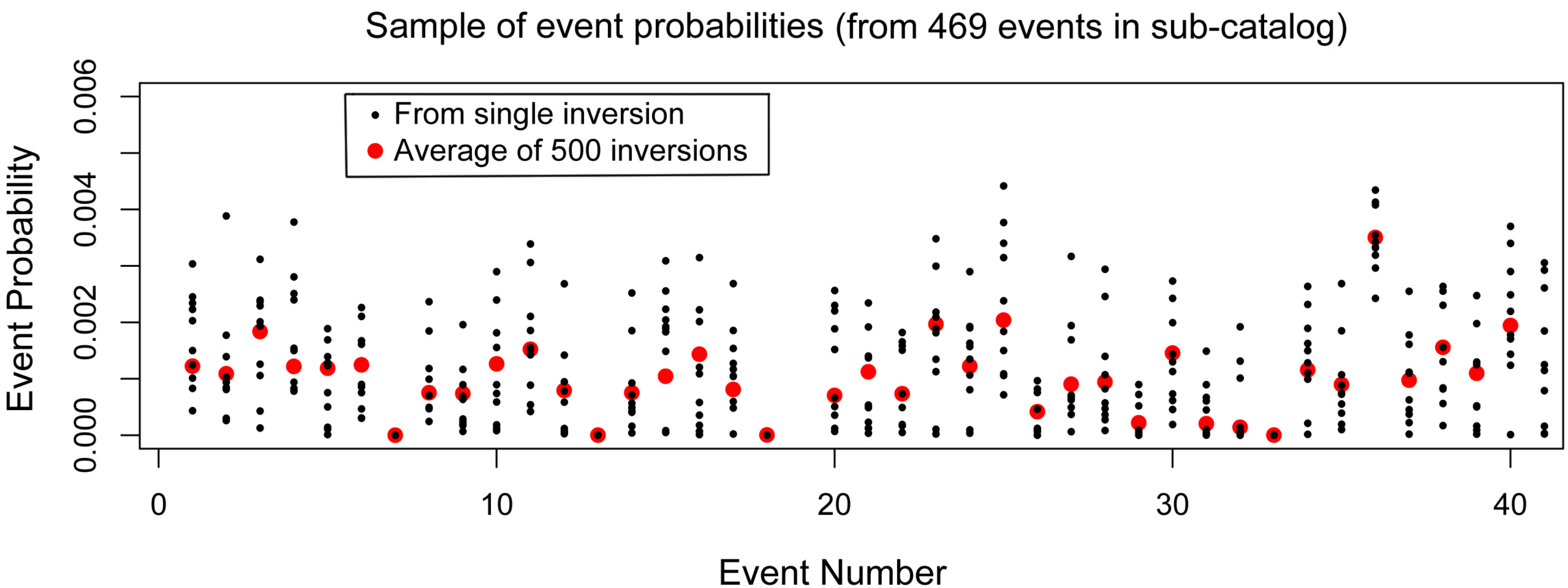
Association function

$$\delta_{ij} = 1 \quad \text{if} \quad \text{slip occurs on section } i \text{ in event } j$$
$$\delta_{ij} = 0 \quad \text{if no slip occurs on section } i \text{ in event } j$$

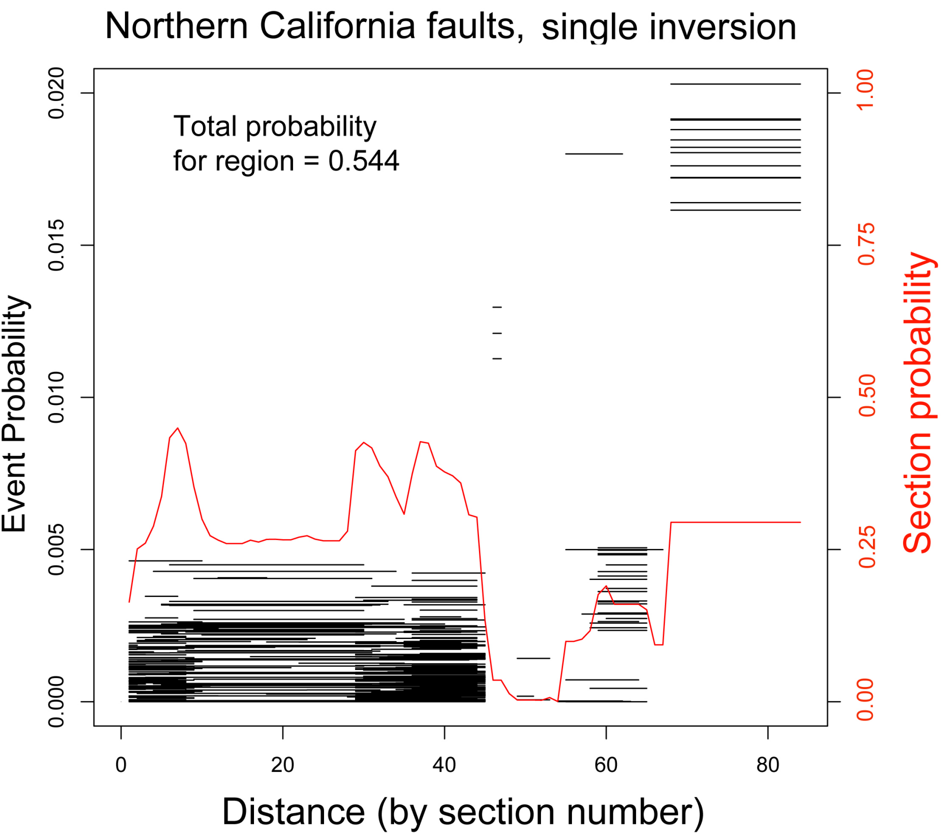
Section number –>

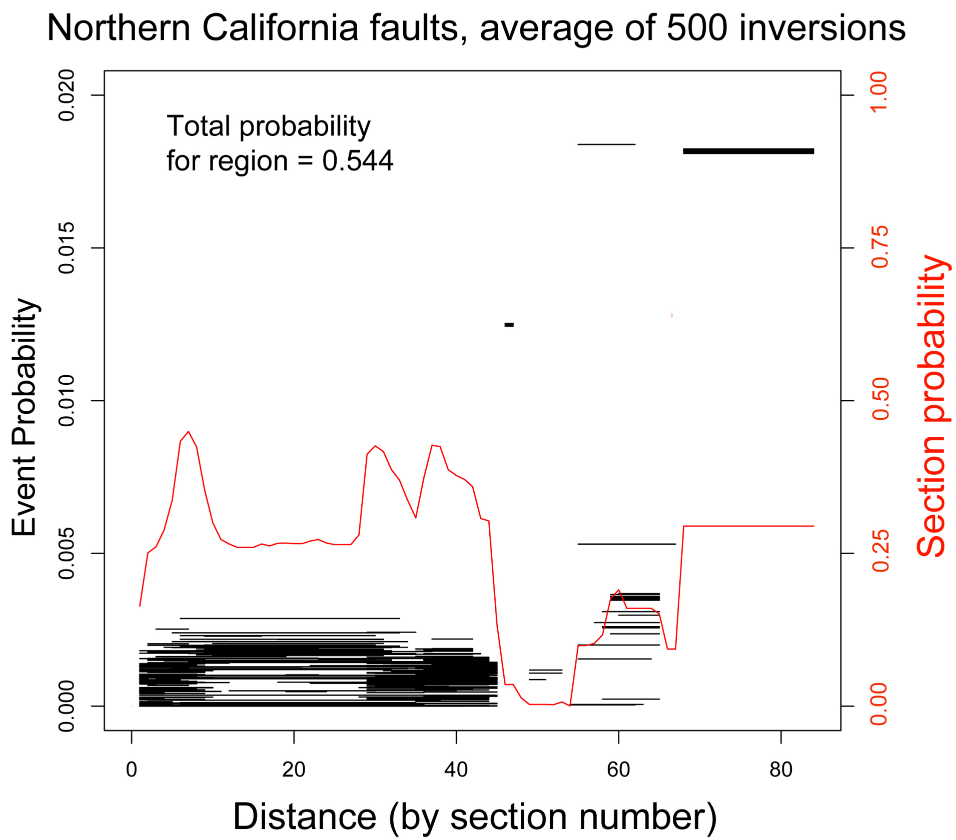
event #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	# sections with event	total sections for event	factor W
1	1	1	1	1	1	1	1	1	1	1	1	1	1			13	13	1.00
2													1			1	13	13.00
3			1	1	1											3	3	1.00
4				1								1		1	1	4	3	0.75
5							1	1								2	2	1.00
6									1							1	1	1.00
7											1					1	3	3.00
8			1													1	1	1.00
9										1				1		2	3	1.50
10	1	1								1	1	1	1	1		7	14	2.00
11	1	1		1	1	1										5	5	1.00
12										1		1				2	2	1.00
13		1	1													2	3	1.50
14						1	1									2	2	1.00
15				1												1	1	1.00
16										1						1	1	1.00
17													1			1	2	2.00
18									1			1	1	1	1	5	5	1.00
19		1	1	1	1											4	6	1.50
20							1									1	1	1.00
21				1												1	1	1.00
22								1								1	2	2.00
23		1	1	1	1											4	4	1.00
24							1	1	1	1						4	3	0.75
25							1	1		1						3	4	1.33

30-year time dependent probabilities
M≥6.7, Northern California faults



Example of 30-yr conditional event probabilities by position in fault system





Earthquake probability for a Region

The events are mutually exclusive (assuming two events cannot occur at the same time) and the event probabilities are independent until the next event happens. At that point the calculation must be re-done. This implies that we can use the standard equation for compounding probabilities to find the probability of any event in Δt.

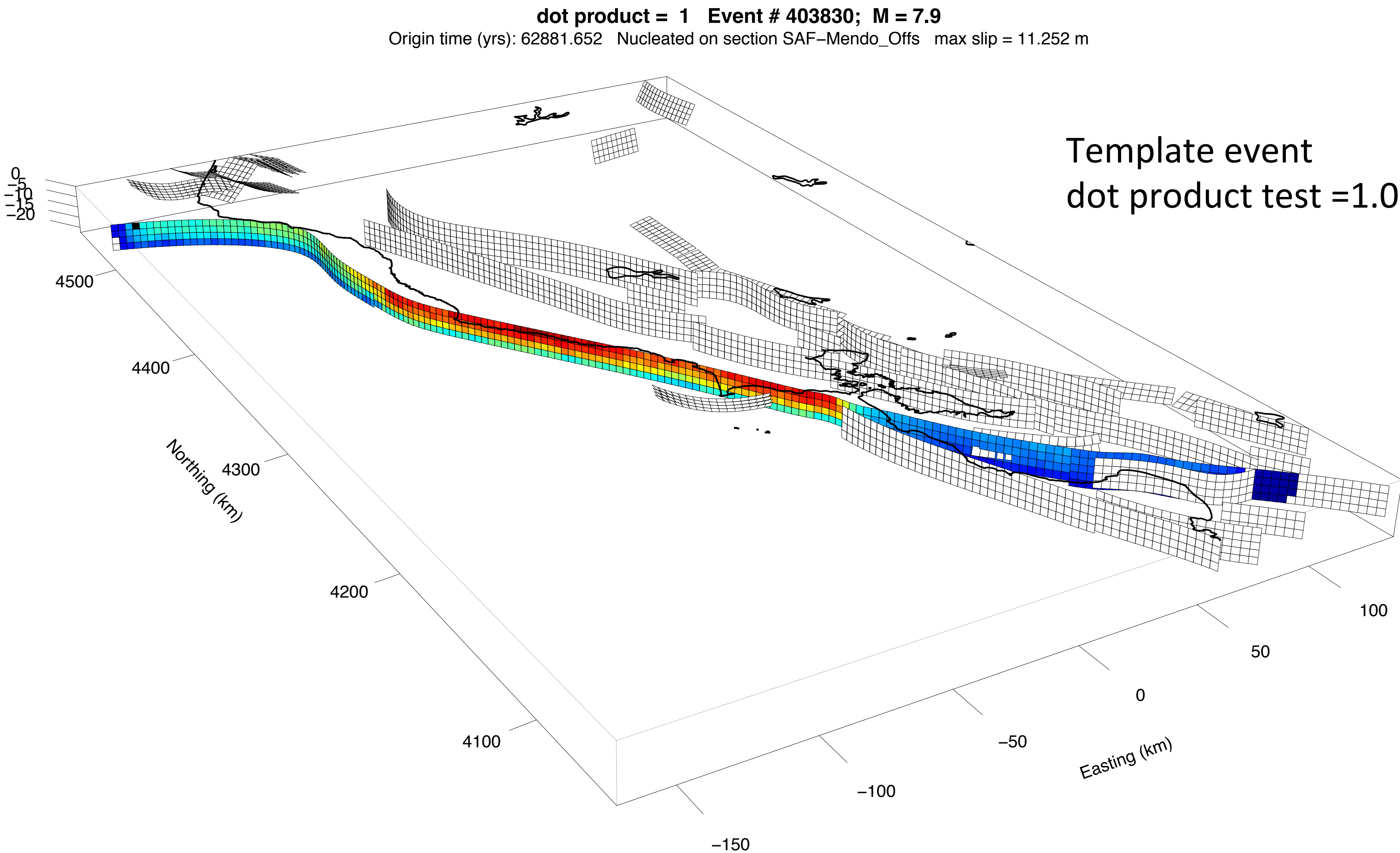
The probability of no event in Δt is

$$P_{ne}=(1-prob_1)(1-prob_2)(1-prob_3) \ldots\ldots$$

and the probability of one or more events is

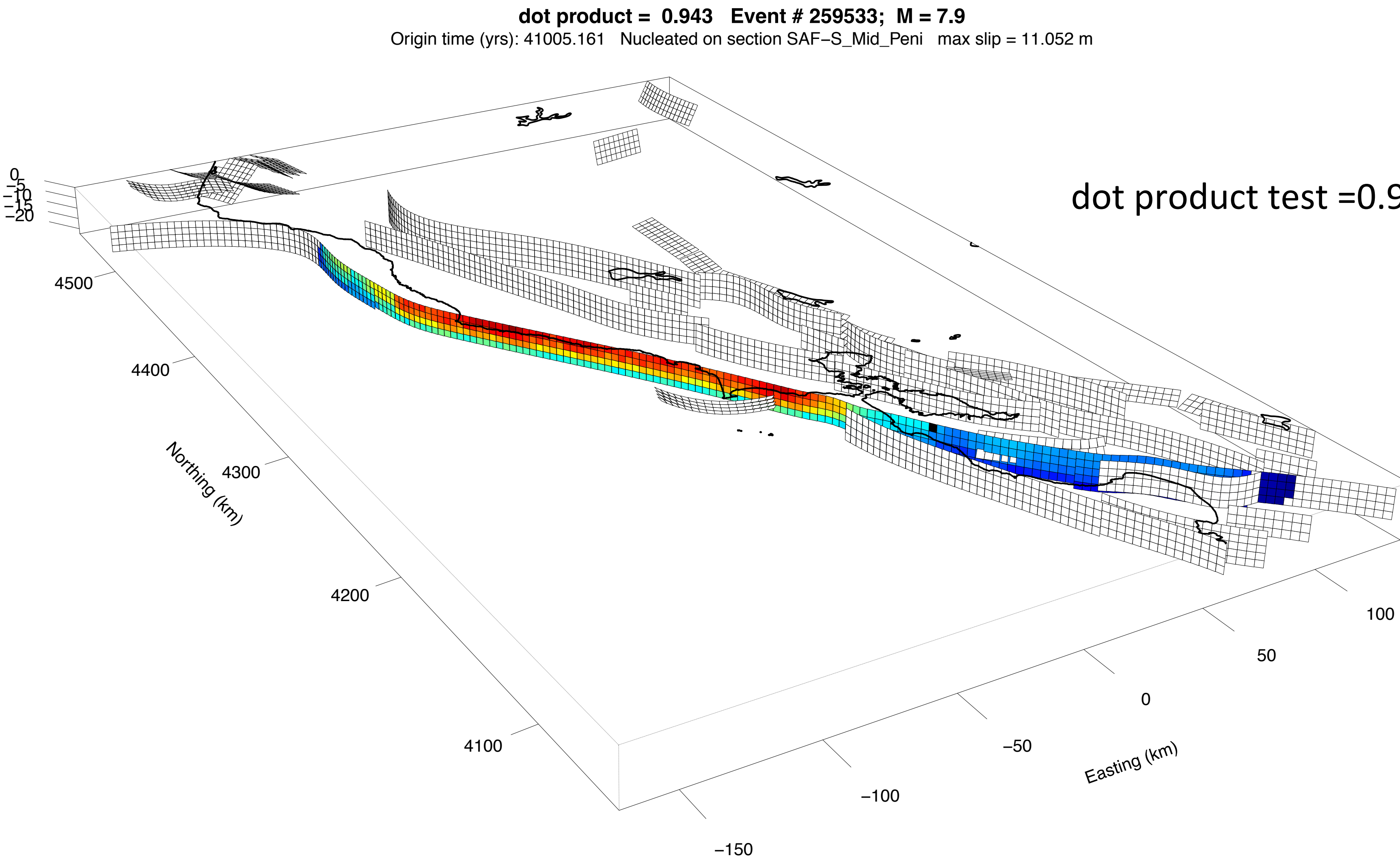
$$P_T=1-P_{ne}=1-(1-prob_1)(1-prob_2)(1-prob_3) \ldots\ldots$$

Rupture Similarity



Template event
dot product test =1.00

Rupture Similarity

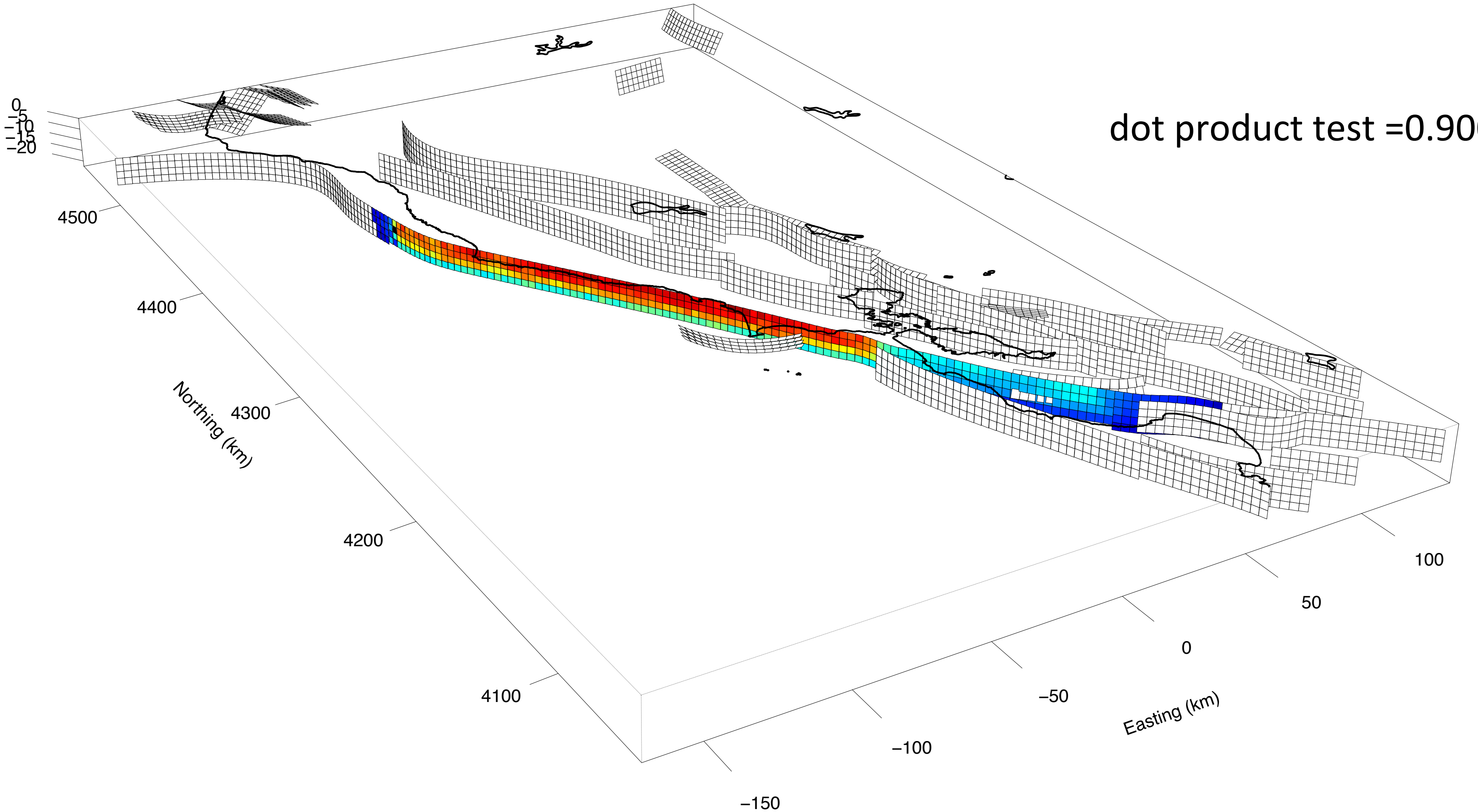


dot product test =0.943

Rupture Similarity

dot product = 0.9 Event # 12221; M = 7.8

Origin time (yrs): 2248.505 Nucleated on section SAF−N_Coast_Of max slip = 9.756 m

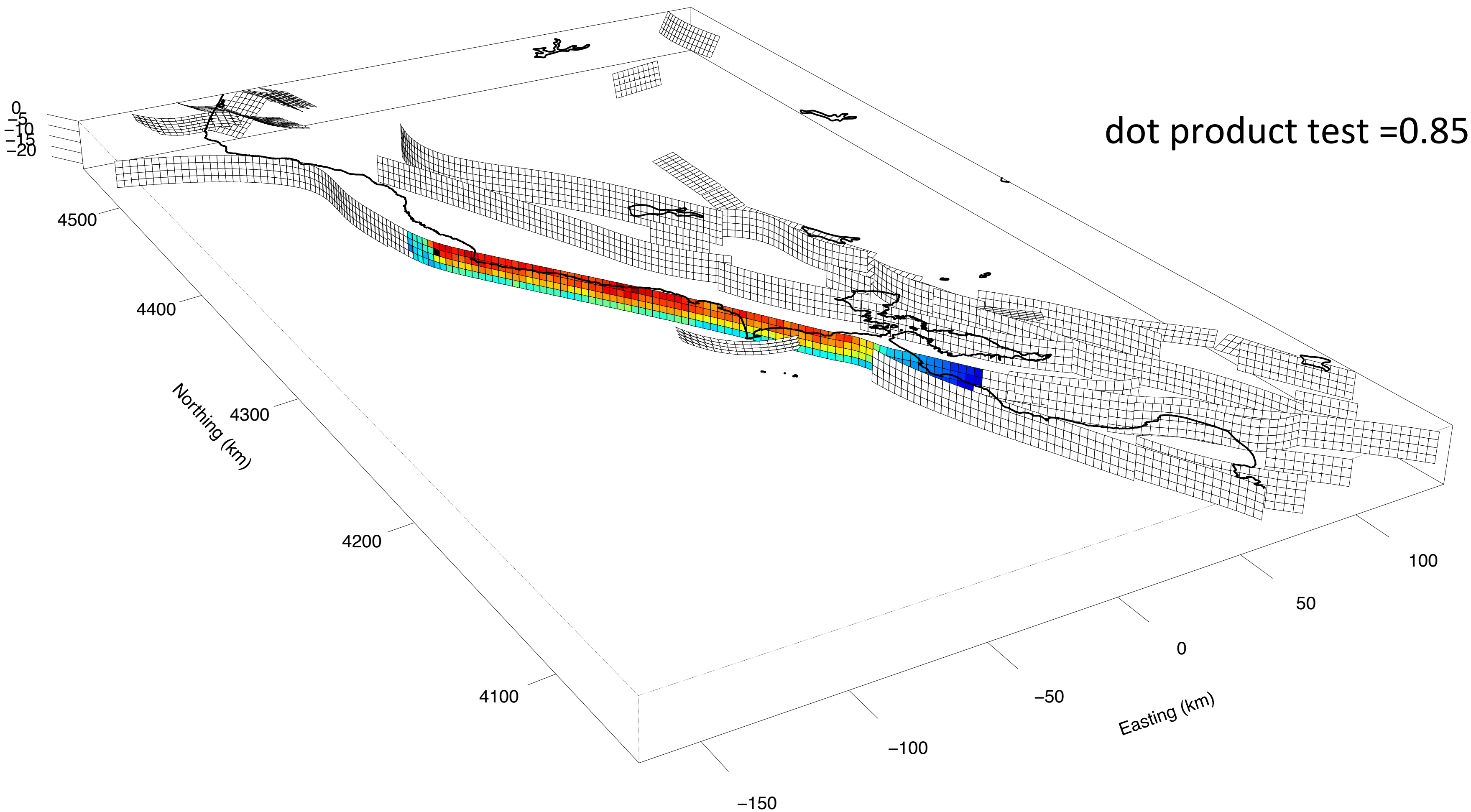


dot product test =0.900

Rupture Similarity

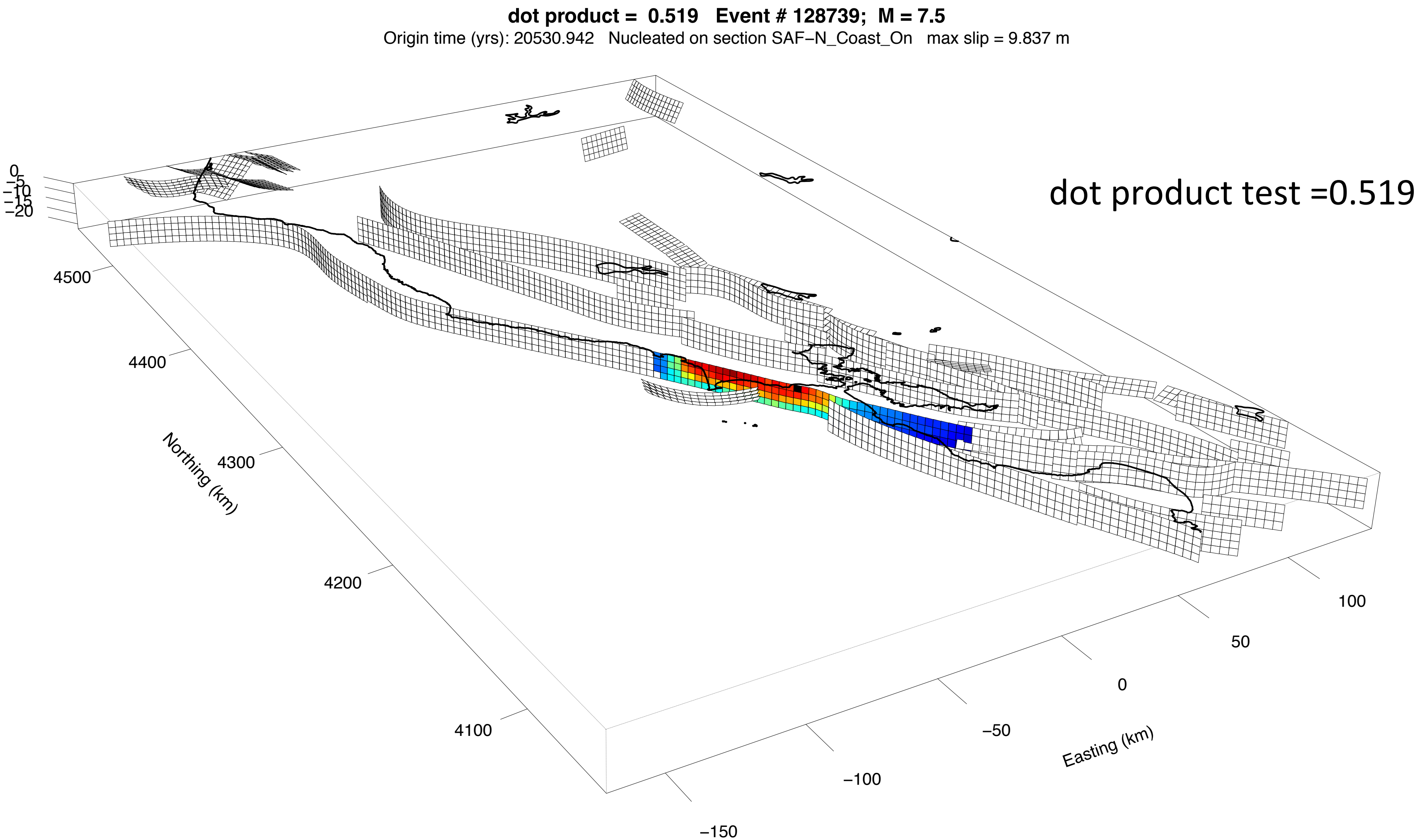
dot product = 0.852   Event # 308793; M = 7.7

Origin time (yrs): 48362.017   Nucleated on section SAF−N_Coast_Of   max slip = 9.539 m



dot product test =0.852

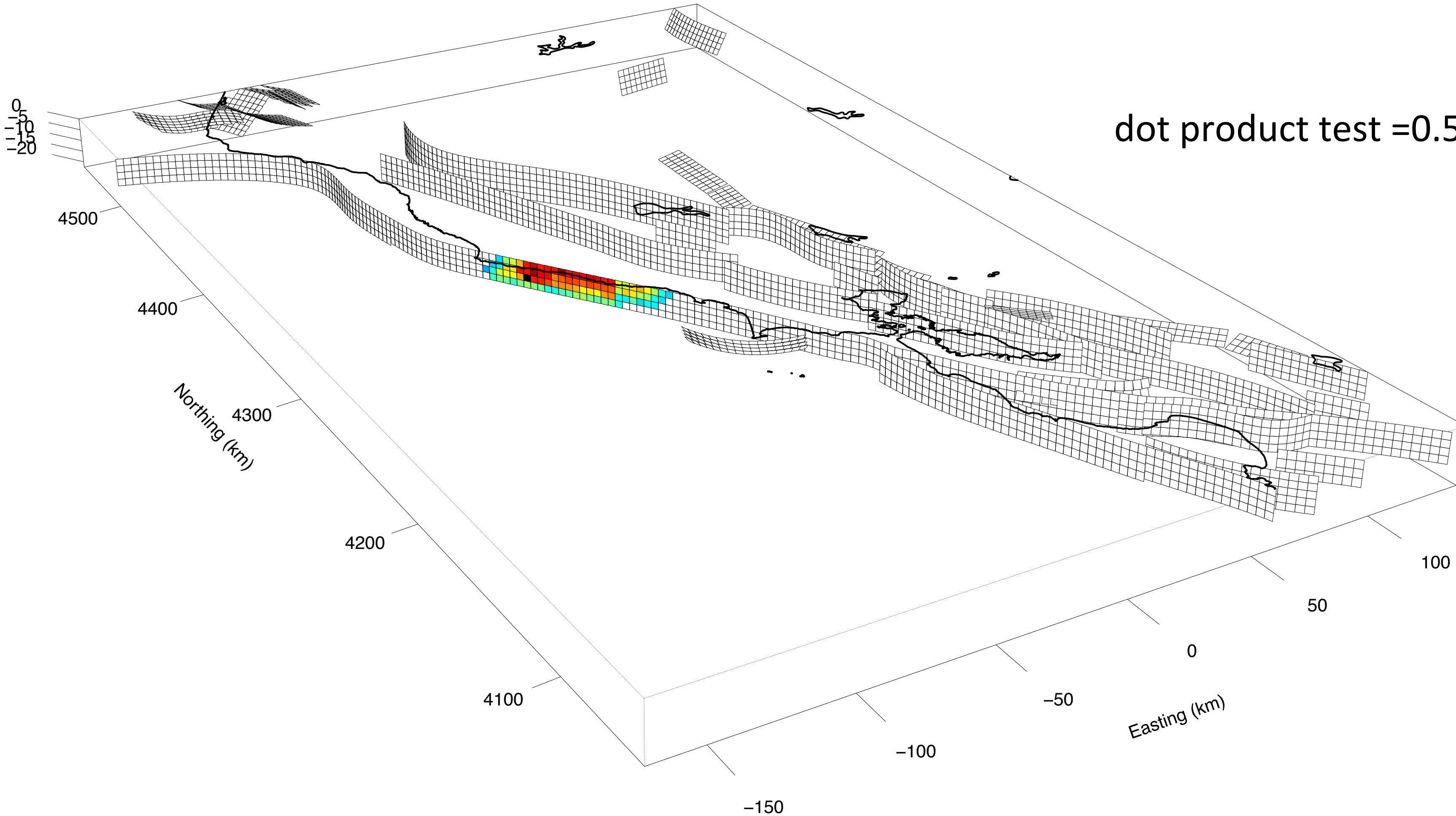
Rupture Similarity



Rupture Similarity

dot product = 0.52 Event # 222276; M = 7.3

Origin time (yrs): 35196.979 Nucleated on section SAF−N_Coast_On max slip = 6.958 m



dot product test =0.520